NOTES ON EXTRACTION OF ENERGY FROM AN EXTREMAL KERR–NEWMAN BLACK HOLE VIA CHARGED PARTICLE COLLISIONS*

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The so-called BSW effect is an idealised scenario for high-energy test particle collisions in the vicinity of black holes; if the black hole is extremal and one of the particles fine-tuned, the centre-of-mass collision energy can be arbitrarily high. It has been recently shown that the energy of escaping particles produced in this process can also be arbitrarily high in the given approximation, as long as both the black hole and the escaping particles are charged, regardless of how small the black-hole charge might be. We revisit these results and show that they are also compatible with properties of microscopic particles for the case of motion in the equatorial plane of an extremal Kerr–Newman black hole.

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1. Introduction

Bañados, Silk, and West (BSW) [1] described a possibility to attain arbitrarily high centre-of-mass collision energies in collisions involving fine-tuned test particles orbiting an extremal Kerr black hole. This BSW effect turned out to be ubiquitous for extremal rotating black holes [2], yet it was shown that the energies of particles that can be produced in such a process are subjected to unconditional upper bounds [3]. On the other hand, no such bounds were found [4] for the variant of the BSW effect that can occur for charged particle collisions in the vicinity of an extremal Reissner–Nordström black hole [5]. Recently, a more general variant of the BSW effect was considered for charged particle collisions near a general extremal rotating electrovacuum black hole [6], unifying the previously known cases. In this setup, it was shown that the unconditional upper bounds on the energy extracted from the black hole by the escaping particles are absent whenever both the black hole and the escaping particles are charged [7].

2. General considerations

Let us first consider a general axially symmetric, stationary electrovacuum black-hole spacetime with metric

$$\boldsymbol{g} = -N^2\,\mathbf{d}t^2 + g_{\varphi\varphi}\left(\mathbf{d}\varphi - \omega\,\mathbf{d}t\right)^2 + g_{rr}\,\mathbf{d}r^2 + g_{\vartheta\vartheta}\,\mathbf{d}\vartheta^2\,, \tag{1}$$

and with the following potential of the electromagnetic field:

$$\boldsymbol{A} = A_t\,\mathbf{d}t + A_\varphi\,\mathbf{d}\varphi = -\phi\,\mathbf{d}t + A_\varphi\left(\mathbf{d}\varphi - \omega\,\mathbf{d}t\right)\,. \tag{2}$$

The horizon of the black hole corresponds to $N = 0$. We further assume that $g_{\varphi\varphi} > 0$, that the product $N\sqrt{g_{rr}}$ is finite and nonvanishing for $N \to 0$, and also that the spacetime has a reflection symmetry with respect to the equatorial "plane" $\vartheta = \pi/2$. Then, we can consider a motion of charged test particles confined to this hypersurface, with the following equations of motion:

$$p^t = \frac{\mathcal{X}}{N^2}\,, \qquad p^\varphi = \frac{\omega\mathcal{X}}{N^2} + \frac{L - qA_\varphi}{g_{\varphi\varphi}}\,, \qquad p^r = \frac{\sigma\mathcal{Z}}{N\sqrt{g_{rr}}}\,. \tag{3}$$

Here, the auxiliary functions $\mathcal{X}$ and $\mathcal{Z}$ are defined by

$$\mathcal{X} = E - \omega L - q\phi\,, \qquad \mathcal{Z} = \sqrt{\mathcal{X}^2 - N^2\left[m^2 + \frac{\left(L - qA_\varphi\right)^2}{g_{\varphi\varphi}}\right]}\,; \tag{4}$$

and E is the particle's energy, L its angular momentum, q its charge, and m its mass. Parameter $\sigma = \pm 1$ controls the direction of the radial motion. In order to preserve causality, we need to impose the condition of motion forward in time, *i.e.*, $\mathcal{X} > 0$.

For an extremal black hole with its horizon located at $r = r_{\mathrm{H}}$, we can write $N^2 = (r - r_{\mathrm{H}})^2 \mathcal{N}^2$. We shall denote values of various quantities at $r = r_{\mathrm{H}}$ with a subscript or superscript H. Let us also define the first-order expansion coefficients, $\hat{\omega}$ and $\hat{\phi}$, of ω and ϕ by

$$\omega = \omega_{\mathrm{H}} + \hat{\omega}\,(r - r_{\mathrm{H}}) + \ldots\,, \qquad \phi = \phi_{\mathrm{H}} + \hat{\phi}\,(r - r_{\mathrm{H}}) + \ldots \tag{5}$$

We can observe that only particles with $\mathcal{X}_{\mathrm{H}} > 0$, called usual, can fall into the black hole. Particles with $\mathcal{X}_{\mathrm{H}} = 0$ are called critical, as they are on the edge of being able to fall inside. For critical particles, we can approximate functions $\mathcal{X}$ and $\mathcal{Z}$ close to r_{H} as follows:

$$\mathcal{X} \approx \chi\,(r - r_{\mathrm{H}}) + \ldots\,, \qquad \mathcal{Z} \approx \zeta_{\mathrm{cr}}\,(r - r_{\mathrm{H}}) + \ldots \tag{6}$$

We can also consider so-called nearly critical particles, which behave approximately as critical at a given radius r_{C}. For such particles we define a formal expansion of $\mathcal{X}_{\mathrm{H}}$ in powers of $(r_{\mathrm{C}} - r_{\mathrm{H}})$,

$$\mathcal{X}_{\mathrm{H}} \approx -C\,(r_{\mathrm{C}} - r_{\mathrm{H}}) - D\,(r_{\mathrm{C}} - r_{\mathrm{H}})^2 + \ldots \tag{7}$$

Then the functions $\mathcal{X}$ and $\mathcal{Z}$ have the following expansions at r_{C}:

$$\mathcal{X} \approx (\chi - C)\,(r_{\mathrm{C}} - r_{\mathrm{H}}) + \ldots\,, \qquad \mathcal{Z} \approx \zeta_{\mathrm{nc}}\,(r_{\mathrm{C}} - r_{\mathrm{H}}) + \ldots \tag{8}$$

Here, χ and ζ_{nc} can be expressed as

$$\chi = -\frac{\hat{\omega}E}{\omega_{\mathrm{H}}} - q\frac{\omega_{\mathrm{H}}\hat{\phi} - \hat{\omega}\phi_{\mathrm{H}}}{\omega_{\mathrm{H}}}\,, \qquad \zeta_{\mathrm{nc}} = \sqrt{(\chi - C)^2 - \mathcal{N}_{\mathrm{H}}^2\left[m^2 + \frac{\left(E + qA_t^{\mathrm{H}}\right)^2}{g_{\varphi\varphi}^{\mathrm{H}}\omega_{\mathrm{H}}^2}\right]}\,, \tag{9}$$

whereas ζ_{cr} is obtained by putting $C = 0$ in ζ_{nc}.

Let us now consider a collision of two incoming particles, a critical particle 1 and a usual particle 2, at a point r_{C} close to r_{H}. It can be shown that the centre-of-mass collision energy E_{CM} in such a process goes like $E_{\mathrm{CM}}^2 \sim (r_{\mathrm{C}} - r_{\mathrm{H}})^{-1}$ (see [6, 7]). Let us assume that two new particles, 3 and 4, are produced in the event. Now, we shall determine whether the arbitrarily high E_{CM} can lead to one of their energies being high, too. Let us first note that $N^2 p^t \mp N\sqrt{g_{rr}}p^r = \mathcal{X} \mp \sigma\mathcal{Z}$. For usual particles, it holds

$\mathcal{X} - \mathcal{Z} \sim (r - r_{\mathrm{H}})^2$ and $\mathcal{X} + \mathcal{Z} \approx 2\mathcal{X}_{\mathrm{H}}$, whereas for (nearly) critical particles, we have $\mathcal{X} \mp \mathcal{Z} \sim (r_{\mathrm{C}} - r_{\mathrm{H}})$. Therefore, if we calculate the sum of the momentum conservation laws for p^t and p^r with appropriate coefficients,

$$N^2 \left(p_1^t + p_2^t\right) \mp N\sqrt{g_{rr}} \left(p_1^r + p_2^r\right) = N^2 \left(p_3^t + p_4^t\right) \mp N\sqrt{g_{rr}} \left(p_3^r + p_4^r\right) , \quad (10)$$

incoming usual particles, outgoing usual particles, and (nearly) critical particles will each have a different leading-order contribution to its expansion in powers of $(r_{\mathrm{C}} - r_{\mathrm{H}})$. We can then infer from (10) with its upper sign that one of the produced particles, say 4, has to be usual and incoming, and hence bound to fall into the black hole. The other produced particle needs to be nearly critical, as required by (10) with its lower sign, which turns into

$$\chi_1 - \zeta_1^{\mathrm{cr}} = \chi_3 - C_3 + \sigma_3 \zeta_3^{\mathrm{nc}} . \quad (11)$$

One can define a new parameter A_1 by $\mathcal{N}_{\mathrm{H}} A_1 \equiv \chi_1 - \zeta_1^{\mathrm{cr}}$ and then solve (11) for C_3 and σ_3. The solution for C_3 can be factorised

$$C_3 = -\frac{\mathcal{N}_{\mathrm{H}}}{2 g_{\varphi\varphi}^{\mathrm{H}} \omega_{\mathrm{H}}^2 A_1} \left(E_3 - R_+\right) \left(E_3 - R_-\right) \quad (12)$$

in terms of $R_\pm(q_3)$. If we choose $R_+ > R_-$, particle 3 will have $C_3 > 0$ for $R_+ > E_3 > R_-$. Since particle 3 with $C_3 > 0$ cannot fall into the black hole, it is locally guaranteed to escape. It can be shown that $R_+(q_3)$ is *generically* the highest energy that a particle with a given value of q_3 can extract from the black hole (see [7]).

3. Selected results for Kerr–Newman solution

The well-known Kerr–Newman solution describes a charged, rotating black hole with mass M, angular momentum aM, and charge Q. The extremal case is defined by the constraint $M^2 = Q^2 + a^2$, which leads to $r_{\mathrm{H}} = \sqrt{Q^2 + a^2}$. The condition $\mathcal{X}_{\mathrm{H}} = 0$ for critical particles implies the following relation among E, L, and q:

$$E \left(Q^2 + 2a^2\right) - aL - qQ\sqrt{Q^2 + a^2} = 0 . \quad (13)$$

Critical particles can participate in near-horizon collisions only when their parameters have values for which the quantity ζ_{cr} is real. Such values form an admissible region in the parameter space of critical particles (see [6, 7]). Its border is given by the condition $\zeta_{\mathrm{cr}} = 0$, which can be expressed as

$$q_\pm(E) = \frac{\sqrt{Q^2 + a^2}}{Q^3} \left[E \left(Q^2 - a^2\right) \pm |a| \sqrt{E^2 \left(Q^2 + a^2\right) - m^2 Q^2}\right] . \quad (14)$$

Plugging in $E = m$, we obtain two values

$$q_+^{\rm mb} = m\frac{\sqrt{Q^2+a^2}}{Q}\,, \qquad q_-^{\rm mb} = m\frac{\sqrt{Q^2+a^2}}{Q^3}\left(Q^2-2a^2\right)\,. \tag{15}$$

Together with the corresponding value of L calculated from (13), $q_+^{\rm mb}$ implies $\mathcal{Z} \equiv 0$. Hence, the motion is allowed only for values in the vicinity of $q_+^{\rm mb}$.

The conditional bound $R_+(q_3)$ on extracted energy is given by

$$\begin{aligned} R_+ \;=\; & \frac{q_3 Q}{\sqrt{Q^2+a^2}} + \frac{1}{Q^2+a^2}\Bigg[2a^2 A_1 \\ & +|a|\sqrt{\left(3a^2-Q^2\right)A_1^2 + 2q_3 Q\sqrt{Q^2+a^2}A_1 - \left(Q^2+a^2\right)m_3^2}\Bigg]\,. \end{aligned} \tag{16}$$

The parameter A_1 takes the form of

$$A_1 = 2E_1 - q_1\tilde{Q} - \sqrt{\left(2E_1 - q_1\tilde{Q}\right)^2 - \left[m_1^2 + \frac{Q^2+a^2}{a^2}\left(E_1 - q_1\tilde{Q}\right)^2\right]}\,, \tag{17}$$

where $\tilde{Q}$ is the specific charge of the black hole defined by $Q = \tilde{Q}\sqrt{Q^2+a^2}$.

4. Conclusions on microscopic particles

In the case of collisions of particles moving along the axis of symmetry of an extremal Kerr–Newman black hole, it was shown [8] that even in the absence of unconditional upper bounds on the extracted energy, the process might still be unviable. In particular, it was found that for $|\tilde{Q}| \ll 1$, only highly relativistic critical particles were able to approach $r_{\rm H}$. In the present setup, however, such a problem clearly does not occur, since the admissible region contains points with $E < m$ whenever $a \neq 0$, as readily seen from (14). Other caveats studied in [8] stemmed from the fact that all charged microscopic or elementary particles known in nature have $|q| \gg m$, and hence the condition (13) generically also implies the initial critical particle 1 to be highly relativistic. In the equatorial case, this issue can be easily circumvented by considering an uncharged particle 1, which is very restrictive, however. Thus, we should examine whether $|q| \gg m$ can be consistent with a nonrelativistic particle 1.

We see that both $q_+^{\rm mb}$ and $q_-^{\rm mb}$ of (15) admit $|q| \gg m$, but only for $|\tilde{Q}| \ll 1$. It is also clear that for $|q| \gg m$, $q_+^{\rm mb}$ implies $\operatorname{sgn} q = \operatorname{sgn} Q$, whereas $q_-^{\rm mb}$ implies $\operatorname{sgn} q = -\operatorname{sgn} Q$. We can expand (14) for $|\tilde{Q}| \ll 1$ and use $E \leqslant m$ to obtain two bounds corresponding to $q_\pm^{\rm mb}$; $q\tilde{Q} < m$ and $-q\tilde{Q}^3 \leqslant 2m$. Since charges of microscopic particles are invariable constants of nature, these expressions can be understood as upper bounds on $|\tilde{Q}|$ of a black hole which permits processes involving critical microscopic particles with $E \leqslant m$.

Starting with the vicinity of q_+^{mb}, let us now expand $\mathcal{A}_1$ of (17) for $E \approx m$ and $|\tilde{q}_1| \sim |\tilde{Q}|^{-1} \gg 1$ (using $q_1 = \tilde{q}_1 m_1$) to obtain

$$\mathcal{A}_1 \approx 2m_1 - q_1\tilde{Q} - \sqrt{2m_1\left(m_1 - q_1\tilde{Q}\right)} . \tag{18}$$

We can infer $\mathcal{A}_1 < m_1$ by utilising $q_1\tilde{Q} < m_1$. Expanding $\mathcal{R}_+$ (16) for $|\tilde{Q}| \ll 1$ and using $\mathcal{A}_1 < m_1$ therein, we get

$$\mathcal{R}_+ \approx q_3\tilde{Q} + 2m_1 + \sqrt{3m_1^2 + 2q_3\tilde{Q}m_1 - m_3^2} . \tag{19}$$

If we take $q_1\tilde{Q} < m_1$ as an upper bound for $|\tilde{Q}|$, this changes to

$$\mathcal{R}_+ \approx m_1\left(\frac{q_3}{q_1} + 2\right) + \sqrt{m_1^2\left(2\frac{q_3}{q_1} + 3\right) - m_3^2} . \tag{20}$$

We see that we can make $\mathcal{R}_+$ large only by requiring $q_3 \gg q_1$. However, for microscopic particles, it is natural to assume the contrary, $q_1 = q_3$, and hence no dependence of $\mathcal{R}_+$ on the charges. If we also put $m_3 = m_1 = m$ for simplicity, we obtain $\mathcal{R}_+ \approx 5m$.

Turning to q_-^{mb}, let us expand $\mathcal{A}_1$ (17) for $|\tilde{q}_1| \gg |\tilde{Q}|^{-1} \gg 1$,

$$\mathcal{A}_1 \approx -q_1\tilde{Q} , \tag{21}$$

and plug it into $\mathcal{R}_+$ (16) expanded for $|\tilde{Q}| \ll 1$ to obtain

$$\mathcal{R}_+ \approx \tilde{Q}\left[q_3 - q_1\left(2 + \sqrt{3 - 2\frac{q_3}{q_1}}\right)\right] . \tag{22}$$

Using $-q_1\tilde{Q}^3 \leqslant 2m_1$ as an upper bound for $|\tilde{Q}|$ and assuming $|q_3| = |q_1| = q$ and $q_3 = -q_1$, this turns into $\mathcal{R}_+ \approx \sqrt[3]{2q^2m_1}(3+\sqrt{5})$. Unlike in the previous case, the bound did not lose the dependence on q and it is consistent with $E_3 \gg m_3$, *i.e.*, with significant extraction of energy from the black hole.

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