

## TECHNIQUE FOR DETERMINATION OF PHASE CHANGES IN MOIST AIR FLOW IN A BLADE CASCADE

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### ABSTRACT

The paper deals with theoretical and numerical tool for prediction of phase changes in moist air flow in flow systems. Partially solidified or liquidized steam in moist air appears to be a problem for operation wind tunnels, turbomachines, etc. Determination of phase change conditions is based on thermodynamic theory. Parameters of saturated homogeneous moist air are solved when expansion process is simulated at calculations. Conditions of saturation are successively verified and finally determined. Special phase diagram for moist air is designed.

### NOMENCLATURE

$c_p$ (J/kg/K)	specific heat capacity at constant pressure
M (1)	Mach number
p (Pa)	pressure
$p_v$ (Pa)	partial pressure of water vapour in moist air
$p''$ (Pa)	saturation pressure of water
r (J/kg/K)	specific gas constant
T (K)	absolute temperature
x (kg <sub>w</sub> /kg <sub>d.a.</sub> )	specific humidity
$\varphi$ (1)	relative humidity of moist air
$\kappa$ (1)	ratio of heat capacities

### Indices

d.a.	dry air
m.a.	moist air
H <sub>2</sub> O	water substance
sat	conditions of saturation

### INTRODUCTION

Low humidity amount in dry air is an important requirement for high-speed experimental aerodynamic research. It is so that to avoid condensation or solidification of partially contained steam in air during expansion. The task of drying of

air is solved in the Aerodynamic laboratory of the Institute of Thermomechanics of Czech Academy of Sciences by means of the special dryer, see Position 1 in Fig.1. Part of humidity from drawn atmospheric air is captured in dry silica-gel. Nevertheless further expansion of air in the Laval nozzle (Position 4, in Fig.1) causes considerable decrease of temperature. For instance about this decrease is 17 per cent of total temperature at Mach number 1, it is 45 per cent of total temperature at Mach number 2, and 55 per cent of total temperature at Mach number 2.5. There is experience that in the case of little dried air, fog or small particles of solid water can appear in high speed air flow in the test section of wind tunnel (Position 5, in Fig.1).

The aim of this paper is to prepare a tool for prediction of saturated moist air during its expansion. Thermodynamics of moist air is a basis for solution of this task. Application of presented results can be not only at high-speed wind tunnels but also at pipeline operation or at gas turbine icing explanation.

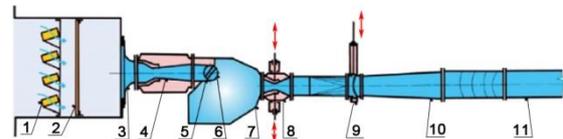


Figure1: Scheme of the indraft type high-speed wind tunnel: 1-silica gel drier, 2-filters, 3-entrance nozzle, 4-inlet nozzle, 5-transient insert, 6-rotable test section, 7-settling chamber, 8-control nozzle, 9-quick acting valve, 10-diffuser, 11-main duct

### THERMODYNAMIC THEORY OF MOIST AIR

Basic parameters are:

total temperature of moist air T [K]

and

total pressure of moist air p [Pa]

They are measured in front of Borda entrance nozzle (Position 3, in Fig.1) of the High-speed wind tunnel. Further important measured parameter should be relative humidity of this air  $\varphi$ .

Specific humidity of moist air is given by equation (1)

$$x = 0,622 \cdot \frac{p_v}{p - p_v} \quad (1)$$

where  $p_v$  is partial pressure of water vapour in moist air, the constant 0,622 is the ratio of molecular weight of water to that of dry air. Partial pressure of water vapour is solved by equation

$$p_v = \phi \cdot p''(T) \quad (2)$$

where  $\phi$  is relative humidity as mentioned above, and  $p''$  is saturation pressure of water depending on given temperature  $T$ .

Formulation IAPWS-IT97 [2] determines implicit quadratic equation from which it is possible to solve relation of pressure of saturated steam (or water liquid) on its temperature  $T$  in the region from triple point to critical point, i.e. for evaporation or condensation, and vice versa

$$\beta^2 \vartheta^2 + n_1 \beta^2 \vartheta + n_2 \beta^2 + n_3 \beta \vartheta^2 + n_4 \beta \vartheta + n_5 \vartheta + n_6 \vartheta^2 + n_7 \vartheta + n_8 = 0 \quad (3)$$

where

$$\beta = \left( \frac{p''}{p^*} \right)^{\frac{1}{4}} \quad (4)$$

$$\vartheta = \frac{T}{T^*} + \frac{n_9}{\frac{T}{T^*} - n_{10}} \quad (5)$$

when  $p^* = 1$  MPa,  $T^* = 1$  K, and constants in Eqs.(3) and (5) are introduced in Tab.1.

Table 1: The constants in Eqs.(3) and (5) [2]

$n_1$	$0.11670521452767 \cdot 10^4$	$n_6$	$0.14915108613530 \cdot 10^2$
$n_2$	$-0.72421316703206 \cdot 10^6$	$n_7$	$-0.48232657361591 \cdot 10^4$
$n_3$	$-0.17073846940062 \cdot 10^2$	$n_8$	$0.40511340542057 \cdot 10^6$
$n_4$	$0.12020824702470 \cdot 10^5$	$n_9$	$-0.23855557567849$
$n_5$	$-0.32325550322333 \cdot 10^7$	$n_{10}$	$0.65017534844798 \cdot 10^3$

Saturation between solid and gas phase of  $H_2O$ , i.e. for sublimation and desublimation, pressure is solved in dependence on temperature  $T$  in region from 50 K to tripple point  $T_{t.p.}$  according to equation (3)

$$p'' = p_{t.p.} \cdot \exp \left( \theta^{-1} \cdot \sum_{i=1}^3 a_i \cdot \theta^{b_i} \right) \quad (6)$$

where

$$\theta = \frac{T}{T_{t.p.}} \quad (7)$$

when  $T_{t.p.} = 273,16$  K,  $p_{t.p.} = 611,657$  Pa and constants in Eq.(6) are introduced in Tab.2.

Table 2: The constants in Eq.(6) [3]

$a_1$	$0.11670521452767 \cdot 10^4$	$b_1$	$0.14915108613530 \cdot 10^2$
$a_2$	$-0.72421316703206 \cdot 10^6$	$b_2$	$-0.48232657361591 \cdot 10^4$
$a_3$	$-0.17073846940062 \cdot 10^2$	$b_3$	$0.40511340542057 \cdot 10^6$

Adiabatic process in moist air at constant specific humidity  $x = \text{const.}$  is described in [4]. Specific gas constant of moist air  $r_{m.a.}$  is solved from relation

$$r_{m.a.} = \frac{1}{1+x} r_{d.a.} + \frac{x}{1+x} r_{H_2O} \quad (8)$$

where specific gas constant of dry air is  $r_{d.a.} = 287.12$  J/kg/K, and specific gas constant of water vapour is  $r = 461.526$  J/kg/K. Specific heat capacity at constant pressure of moist air  $c_{p.m.a.}$  is solved from relation

$$c_{p.m.a.} = \frac{1}{1+x} c_{p.d.a.} + \frac{x}{1+x} c_{p.H_2O} \quad (9)$$

where specific heat capacity at constant pressure of dry air is  $c_{p.d.a.} = 1005.9$  J/kg/K, and specific heat capacity at constant pressure of water vapour is  $c_p = 1884$  J/kg/K. The Poisson constant, i.e. ratio of heat capacities, of moist air  $\kappa_{p.m.a.}$  is solved under assumption of ideal gas theory from relation

$$\kappa_{m.a.} = \frac{c_{p.m.a.}}{c_{p.m.a.} - r_{m.a.}} \quad (10)$$

Then adiabatic process in moist air from total (measured) parameters (pressure  $p$ , temperature  $T$ ) to saturated conditions gives pressure of moist air  $p_{m.a.sat.}$  in dependency on temperature  $T_{sat}$  by relation

$$p_{m.a.sat.} = p \cdot \left( \frac{T_{sat}}{T} \right)^{\frac{\kappa_{m.a.}}{\kappa_{m.a.} - 1}} \quad (11)$$

The final state of the expansion process for saturation conditions is defined by relative humidity  $\phi_2 = 1$ . Then partial pressure of water vapour is equal to pressure of saturated steam, see Eq.(2), and pressure of moist air at saturated condition can be solved from Eq.(1) as depending on temperature

$$p_{m.a. sat} = p''(T_{sat}) \cdot \left( \frac{0.622}{x} + 1 \right) \quad (12)$$

Two equations, Eq.(11) and Eq.(12), can be solved at application IAPWS data for phase changes in water vapour, i.e. Eqs.(3) to (5) or (6),(7). Efficient iterative process is proposed and parameters, i.e.

pressure  $p_{m.a. sat}$  and temperature  $T_{sat}$ , of moist air under saturation conditions are solved.

### TECHNIQUE FOR DETERMINATION OF PHASE CHANGES IN MOIST AIR

Calculation software for solution of expansion of moist air is prepared [5] and it can be a support for investigations at high-speed aerodynamic research. Likewise it can be an aid for calculations at design and operation of turbomachines.

Special phase diagram for moist air is designed. In principle it is p-t diagram (phase diagram of moist air) (Fig.2) in which limits of saturated moist air are drawn according to Eq.(12) for constant specific humidity  $x''$ . Curves of constant entropy according to Eq.(11) are shown in the diagram in Fig. 2.

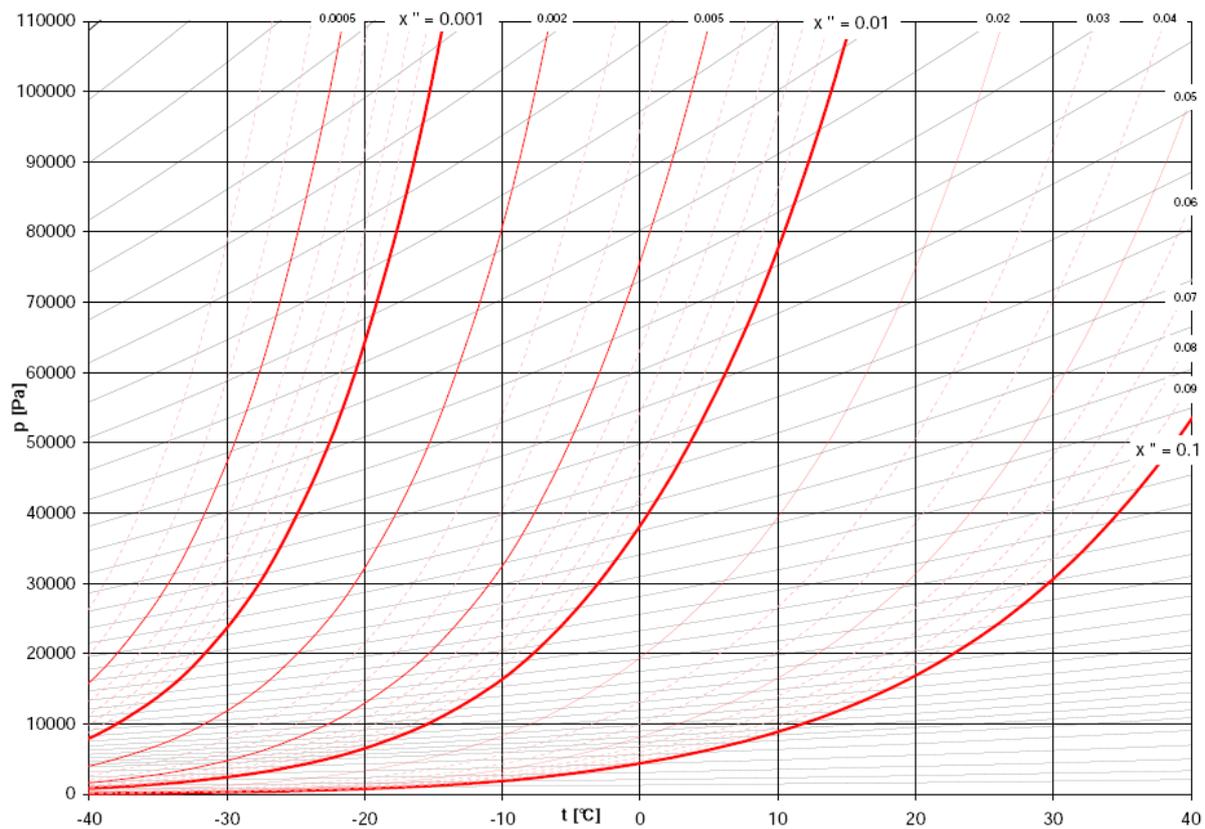


Figure 2: Diagram for evaluation parameters at moist air expansion and limits with saturated moist air. (Units of specific humidity  $x''$  are kg /kg<sub>d.a.</sub>)

In the diagram it is possible to find point determined by total parameters, i.e. pressure  $p$  a temperature  $T$ . The isentrope passing this point intersects proper curve of  $x'' = x$  in the point of saturated moist air at pressure  $p_{m.a. sat}$  and temperature  $T_{sat}$ . It is possible to evaluate limit Mach number at saturation of water vapour in moist air by the Eq.(13)

$$M_{sat} = \sqrt{\frac{2}{\kappa_{m.a.} - 1} \cdot \left[ \left( \frac{p_{m.a. sat}}{p} \right)^{\frac{\kappa_{m.a.}}{\kappa_{m.a.} - 1}} \right]} \quad (13)$$

The technique for determination of phase changes was applied in investigation of influence of phase changes in blade cascade flow in wind tunnel tests on kinetic energy loss coefficient [6].

Rising of kinetic energy loss coefficient was observed in the range of relative humidity  $10\% < \varphi < 25\%$  in transonic region  $0.8 < M_{2is} < 1.4$ . Part of vapour was changed in ice.

### CONCLUSIONS

Theoretical and numerical tool for determination of water vapour changes in expanding moist air is prepared. It can predict parameters at achievement of saturation conditions. Thermodynamic theory of moist air and IAPWS (International Association for Properties of water and Steam) data are a basis for iterative solution process. Special diagram for parameters at expansion of moist air is developed. Proposed technique for determination of saturation conditions should be verified by experiments.

### ACKNOWLEDGMENTS

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